



Fractions

1. If $\frac{8x + 3y}{3y} = \frac{5}{4}$, what is the value of $\frac{x}{y}$?
- A. $\frac{1}{24}$
 - B. $\frac{3}{32}$
 - C. $\frac{5}{16}$
 - D. 3
 - E. $\frac{10}{3}$

OUR APPROACH:

Whenever you see a problem with two fractions set equal to one another, you should consider simplifying the equation by using the ancient art of "cross-multiplication." It works like this:

Ex: If $\frac{a}{b} = \frac{2}{3}$ then $3a = 2b$.

Cross-multiplication helps clean up harder problems too, like the one below!

If you cross-multiply the equation below, you'll get:

$$(8x + 3y)(4) = (3y)(5)$$

Simplify by distributing and you'll get:

$$32x + 12y = 15y$$

Subtract $12y$ from each side and you'll get:

$$32x = 3y$$

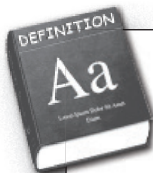
Keep working towards $\frac{x}{y}$ by dividing both sides by 32 .

You'll get:

$$x = \left(\frac{3}{32}\right)y$$

And finally, isolate $\frac{x}{y}$ by dividing both sides by y and you'll get:

$$\frac{x}{y} = \frac{3}{32}$$



FRACTION (N): a part of a whole, represented as the part divided by the whole

Ex: $\frac{1}{4}$

NUMERATOR (N): the top number in a fraction, representing the number of parts taken from the whole

Ex: $\frac{1}{4}$

DENOMINATOR (N): the bottom number of a fraction, representing the number of parts into which the whole is divided

Ex: $\frac{1}{4}$

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TURN AN UGLY DECIMAL INTO A FRACTION!

Simply put parentheses around each fraction and use the division key in place of the fraction bar.

Ex: $\left(\frac{3}{7}\right) \div \left(\frac{12}{5}\right) = (3 \div 7) \div (12 \div 5) = .1785714286$

Your calculator has no respect for the ACT, so it gives you a big ugly decimal. But you can turn that big ugly decimal into a fraction by pressing **MATH**, choosing **FRAC**, then **ENTER**.

$.1785714286 \rightarrow \frac{5}{28}$

KNOWING FRACTIONS IS 1/2 THE BATTLE.

Simplify the following expressions.

1. $\frac{7}{3} + \frac{1}{5} =$

4. $\frac{2}{7} + \frac{9}{7} =$

2. $\frac{\frac{5}{2}}{\frac{3}{5}} =$

5. $\frac{3}{4} - \frac{x}{5} =$

3. $\frac{63}{21} =$

6. $\frac{6n+9}{12n} =$

CONVERT EACH OF THE FOLLOWING INTO A FRACTION.

7. $75\% =$

10. $.333... =$

8. $.6 =$

11. $125\% =$

9. $45\% =$

12. $5\frac{3}{4} =$

AT THIS TIME, PLEASE REMAIN IN YOUR SEAT AND SOLVE EACH OF THE FOLLOWING.

13. If a bus carries 24 boys and 30 girls, what fraction of the passengers is female?

14. If $\frac{3}{4}$ of the seats in a movie theater are occupied, and there are 12 empty seats, how many total seats are there in the theater?15. If it rained $\frac{2}{5}$ of the days in a 120 day period, how many days got no rain at all?



Fractions

1. If $\frac{1}{x} = \frac{3}{y}$, then what is the value of y when $x = 13$?

- A. 3
- B. 9
- C. 13
- D. 24
- E. 39

2. If $\frac{10x + 4y}{3y} = \frac{7}{4}$, what is the value of $\frac{x}{y}$?

F. $\frac{3}{40}$

G. $\frac{1}{8}$

H. $\frac{21}{40}$

J. 2

K. $\frac{8}{3}$

3. If $\frac{7}{a} + \frac{7}{b} + \frac{7}{c} < \frac{7}{a} + \frac{7}{n} + \frac{7}{c}$ and $a > b > c > d$, then n could be which of the following?

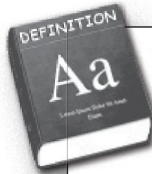
- I. a
- II. c
- III. d

- A. I only
- B. II only
- C. III only
- D. II and III only
- E. I, II and III



Divisibility, Factors & Multiples

1. When the integer n is divided by 10, the remainder is 7. What is the remainder when $2n$ is divided by 5?
 - A. 1
 - B. 2
 - C. 3
 - D. 4
 - E. 5



DIVISIBLE (ADJ): capable of being divided by another integer evenly (without producing a remainder)

FACTOR (N): one of two or more integers that can be multiplied to produce another integer

Factors of 10: 1, 2, 5, and 10

MULTIPLE (N): the product of two integers (factors)

Multiples of 10: 10, 20, 30...

OUR APPROACH:

To solve remainder problems like this one, begin by asking yourself, "Hey self, what's the smallest possible value for n ?" If you're being honest with yourself, you'll have to answer, "17, because the smallest number that gives you a remainder of 7 when divided by 10 is 17."

If you weren't able to help yourself as much as you'd hoped this time around, perhaps next time you'll realize that you can always find the smallest possible value of n by adding the divisor (10 in this case) to the remainder (7 in this case).

Now that we know that n could equal 17, let's plug 17 into the problem in place of n and find the remainder for $2n$ divided by 5.

If $n = 17$ then $2n = 34$. Almost done.

$\frac{34}{5} = 6$ remainder 4.

1. When the integer n is divided by 10, the remainder is 7. What is the remainder when $2n$ is divided by 5?

- A. 1
- B. 2
- C. 3
- D. 4**
- E. 5

$\frac{2(17)}{5} = x$ r. \square

We don't really care about this number.

The remainder is what we're asked to find. This setup is very handy for this type of problem. Take a good look at it here, and practice using it to solve the problems on the next page.

(E) cannot be the answer because the remainder must always be less than the divisor. Otherwise that's like saying $25/5 = 4$ remainder 5. That's 4 groups of 5 and... one more group of 5. If we divided by 5, we can't have 5 left over as a remainder. We'd just have one more group of 5 and no remainder.



HOW TO FIND THE UNKNOWN NUMBER:

Add the divisor and the remainder to find the lowest possible value of the unknown number. Then plug that value into the second expression and solve for the remainder.

For remainder problems, use the setup above and remember, that your answer is not x . You're looking for the remainder!



SOLVE FOR THE REMAINDERS.

1. $\frac{333}{11} =$

4. $\frac{117}{3} =$

2. $\frac{599}{6} =$

5. $\frac{86}{6} =$

3. $\frac{121}{4} =$

6. $\frac{226}{9} =$



THE LEAST COMMON MULTIPLE

Find the least common multiple of the following pairs of numbers.

7. 4 & 6 _____ 11. 6 & 8 _____

8. 4 & 14 _____ 12. 12 & 9 _____

9. 16 & 6 _____ 13. 7 & 13 _____

10. 5 & 3 _____ 14. 24 & 18 _____



WHAT IS THE SMALLEST VALUE OF n ?

15. When n is divided by 6, the remainder is 1.
What is the value of n ?

17. When $2n$ is divided by 6 the remainder is 4.
What is the value of n ?

16. The remainder is 6 when n is divided by 21.
What is the value of n ?

18. If the remainder is 4 when $3n$ is divided by 11, what is the remainder when $5n$ is divided by 4?



Divisibility, Factors & Multiples

1. If as many 7-inch pieces of string as possible are cut from a piece of string that is 11 feet long, what is the length, in inches, of string that is left over?
(12 inches = 1 foot)

A. 2
B. 3
C. 4
D. 5
E. 6

2. How much greater is the remainder when 357 is divided by 4 than the remainder when 243 is divided by 3?

F. 0
G. 1
H. 2
J. 3
K. 4

3. When the integer n is divided by 8, the remainder is 3. What is the remainder when $3n$ is divided by 8?

A. 0
B. 1
C. 2
D. 3
E. 4

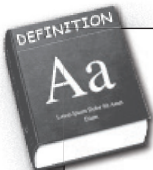
4. The integer n is a multiple of 4 between 21 and 39. Which of the following lists all the possible remainders when n is divided by 7?

F. 0, 2, 4, 6
G. 1, 3, 5, 7
H. 0, 1, 3, 4
J. 1, 2, 3, 4
K. 0, 1, 2, 3, 4, 5, 6, 7



Prime Numbers

1. How many distinct prime factors does the number 120 have?
 - A. 5
 - B. 4
 - C. 3
 - D. 2
 - E. 1



OUR APPROACH:

To solve this problem, make a factor tree and break 120 down by dividing out factors. Keep going until you can no longer divide your factors into smaller numbers. You'll be left with the prime factors of 120. Count them up, but don't double count anything. Then choose your answer..

PRIME (ADJ): describes a number that is larger than 1 and divisible by only 1 and itself

Ex: 2, 3, 5, 7, 11 and 13 are all prime numbers.

FACTOR (N): one of two or more integers whose product is a given integer

Ex: 2 and 3 are factors of 6.
2, 3, 4, 6, 8, and 12 are factors of 24

PRIME FACTOR (N): all the positive prime factors that divide into a given integer exactly, without leaving a remainder:

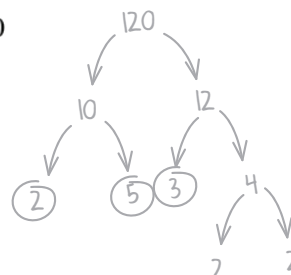
Ex: 2 and 3 are the prime factors of 24.

The word "distinct" is important in this problem. It means "different." So don't double count any of the factors of 120!

1. How many distinct prime factors does the number 120 have?

(A) is a TRAP ANSWER!
If you chose (A) it's probably because you counted the prime factor 2 more than once.

- A. 5
- B. 4
- C. 3**
- D. 2
- E. 1



ONE IS SUCH A LONELY NON-PRIME NUMBER.

It's important to remember that the number 1 is not considered a prime number. That simple fact can affect your score when you have to answer a question like, "How many prime factors of 30 are less than 10?" If you count the number 1 as a prime factor, you're blowing it!

HOT TIP!

THE PRIME SUSPECTS

The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Most (but not all!) prime number problems on the ACT deal with these guys.

It's worth noting that 2 is the ONLY prime number that's not odd.

If you need to check whether a number is prime, just remember that every odd number less than 121 that's not divisible by 3, 5, or 7 is prime.



FIND THE DISTINCT PRIME FACTORS OF EACH OF THE FOLLOWING NUMBERS

1.

30

5.

240

2.

18

6.

660

3.

87

7.

800

4.

96

8.

972



Prime Numbers

1. If the sum of four different prime integers is odd, what is the smallest of the four prime integers?
 - A. 2
 - B. 3
 - C. 5
 - D. 7
 - E. It cannot be determined from the information given.

2. The product of three consecutive prime numbers is 105. What is the greatest of the three numbers?
 - F. 5
 - G. 7
 - H. 9
 - J. 11
 - K. 13

3. If n is the number of distinct positive factors of 30 and p is the number of distinct prime factors of 30, what is the value of $n - p$?
 - A. 5
 - B. 3
 - C. 2
 - D. 1
 - E. 0

4. How many prime numbers are between 0 and 100 inclusive?
 - F. 16
 - G. 19
 - H. 25
 - J. 27
 - K. 31